Behavior of morphological associative memories with true-color image patterns

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A B S T R A C T

Morphological associative memories (MAMs) are a special type of associative memory which exhibit optimal absolute storage capacity and one-step convergence. This associative model substitutes the additions and multiplications used by other models by computing maximums and minimums. This type of associative model has been applied to different pattern recognition problems including face localization and gray scale image restoration. Despite of his power, MAMs have not been applied in problems that involve true-color patterns. In this paper it is described how a MAM can be applied in problems involving true-color patterns. Furthermore, a complete study of the behavior of this associative model in the restoration of true-color images is performed using a benchmark of 14 400 images altered by different type of noises.

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1. Introduction

The concept of associative memory (AM) emerges from psychological theories of human and animals learning. These memories store information by learning correlations among different stimuli. When a stimulus is presented as a memory cue, the other is retrieval as a consequence; this means that the two stimuli have become associated each other in the memory.

An AM can be seen as a particular type of neural network designed to recall output patterns in terms of input patterns that can appear altered by some kind of noise. Several associative models have been described in the last years (refer for example [1–11]). Most of these AMs have several constraints that limit their applicability in complex problems. Among these constraints we could mention their capacity of storage (limited), the type of patterns (only binary, bipolar, integer or real patterns), robustness to noise (additive, subtractive, mixed, Gaussian noise, etc.).

A first attempt in formulating useful morphological neural networks was proposed by Davidson et al. [12]. Since then, only a few papers involving morphological neural networks have appeared. Refer for example to [13,14]. In 1998, Ritter et al. [8] proposed the concept of morphological associative memory (MAM) and the concept of morphological auto-associative memory (MAAM). Basically, the authors substituted the outer product by max and min operations. One year later, the authors introduced their morphological bidirectional associative memories [15]. Their properties, compared with Hopfield Associative model are completely different. For example, they exhibit optimal absolute storage capacity and one-step convergence in the auto-associative case.

This type of associative model has been applied to the reconstruction of gray scale images [9,16–20]. Despite of his power, it has not been applied to problems involving true-color patterns; neither a deep study of this associative model under true-color image patterns has been reported.

In this paper it is described how a MAM can be applied in problems involving true-color patterns. Furthermore, a complete study of the behavior of this associative model in the restoration of true-color images is performed. For this a benchmark of 14 400 images altered by different type of noises is used. In addition, the potential of the described model is tested in two real scenarios: image categorization and image restoration.

2. Basics on morphological associative memories

The basic computations occurring in the morphological network proposed by Ritter et al. are based on the algebraic lattice structure \( (R, \wedge, \vee, +) \) where the symbols \( \wedge \) and \( \vee \) denote the binary operations of minimum and maximum, respectively.

Let \( \mathbf{x} \in \mathbb{R}^n \) and \( \mathbf{y} \in \mathbb{R}^m \) an input and output pattern, respectively. An association between input pattern \( \mathbf{x} \) and output pattern \( \mathbf{y} \) is denoted as \( (\mathbf{x^\zeta}, \mathbf{y^\zeta}) \), where \( \zeta \) is the corresponding association.
In terms of Eqs. (2) and (3), this last set of inequalities implies that
\[ W_{XY} \leq \text{MAM}_{XY} \]
and equivalently,
\[ Y = \text{MAM}_{XY} X \]

The complete set of theorems which guarantee perfect recall and their corresponding proofs are given in [8]. Something important to mention is that this MAM is robust either to additive noise or to subtractive noise, not both (mixed noise). While MAM \( W_{XY} \) is robust to subtractive noise, MAM \( M_{XY} \) is robust to additive noise. However, this MAM in not robust to image transformations which make it inadaptable to be directly applied in object recognition problems.

3. Behavior of \( W_{XY} \) under true-color noisy patterns

In this section a study of the behavior of \( W_{XY} \) under true-color noisy patterns is presented. Two types of experiments will be performed. In the first case we study the auto-associative version of \( W_{XY} \), in the second case we study the hetero-associative version.

For the case of the auto-associative version, first to all, we verified if the MAAM \( W_{XY} \) was capable to recall the complete set of associations. At last, we verified the behavior of \( W_{XY} \) using noisy versions of the images used to train the MAAM.

The benchmark used in this set of experiments is composed by 14 400 color images of 63 x 43 pixels and 24 bits in a bmp format. This benchmark contains 40 classes of flowers and animals. For each class, there are 90 images altered with additive noise (0% of the pixels to 90% of the pixels), 90 images altered with subtractive noise (0% of the pixels to 90% of the pixels), 90 images altered with mixed noise (0% of the pixels to 90% of the pixels) and 90 images altered with Gaussian noise (0% of the pixels to 90% of the pixels). Fig. 1 shows some images which compose this benchmark.

Although it seems that there is not much difference between gray level images and true-color images, MAMs are not designed to cope with multivariable patterns (three channels per pixel) because they are based on gray level morphological operations. Instead of training one memory per color channel and then deciding how to combine the information recalled by each memory and finally restore the true-color image, we proposed to transform these three channels in one channel. However, it is important to notice that if we transform the RGB channels into one channel by means of computing the average of the three channels (in other words transform the true-color image into a gray level image), we will not be able to recover the information of the RGB channels from the average channel.

For that reason, before the MAM \( W_{XY} \) was trained, each image had to be transformed into an image pattern. To build an image pattern from the BMP file, the image was read from left-right and up-down; each RGB pixel (hexadecimal value) was transformed into a decimal value and finally, this information was stored into an array. For example, suppose that the value of a RGB pixel is "0x3E35A1" (where R=3E, G=53 and B=A1) then by transforming into its decimal value, its corresponding decimal value will be "4084641". With this new procedure, we avoid training the MAM with multivariable patterns and we can recover the RGB channels by transforming the decimal value into its hexadecimal value.

Once trained the associative memory, we proceeded to evaluate the behavior of the two MAM \( W_{XY} \) versions. In order to measure the accuracy of the MAM we counted the number of pixels correctly recalled. We reported the percentage of pixels that can be recalled by the method because we wanted to emphasize if the associative model really restores the altered pixels. However, for practical image processing purposes, we also used the normalized mean square error to measure the difference between the original image and the recalled image.

3.1. Auto-associative version of \( W_{XY} \)

It is important to remark that even using true-color patterns the MAAM \( W_{XY} \) was capable to recall the complete set of associations. However, it is important to analyze the robustness that the model presents in the presence of noisy patterns.

In the next set of experiments, we study the behavior of the MAAM \( W_{XY} \) under different type of noises. For the case of additive noise (Fig. 2(a)), we can observe that if only the 2% of the pixels are altered, the MAAM \( W_{XY} \) is capable of correctly recalling only the 23.6% of the pixels. For the case of mixed and Gaussian noise (Figs. 2(c and d)), we can observe that if only the 2% of the pixels are altered, the MAAM \( W_{XY} \) is capable of correctly recalling only the 29.7% and 46.8% of the pixels, respectively. These percentages decrease as the number of altered pixels increases. For the case of subtractive noise (Fig. 2(b)), we can observe that even when the 90% of the pixels are altered, the MAAM \( W_{XY} \) is capable of correctly recalling the 77.4% of the pixels.
In average, for the case of the image patterns altered with additive noise the MAAM $W_{XY}$ recalled 9.28% of the pixels. For the case of subtractive noise the MAAM $W_{XY}$ in average recalled the 87.7% of the pixels. For the case of mixed and Gaussian noise, the MAAM $W_{XY}$ recalled the 10.41% and 16.25%, respectively.

Other important issue that we would like point out is the storage capacity of this associative model. In the previous
experiment we stored 40 associations and we obtained an accuracy of 87.7% for the case of the MAAM $W_{XY}$ with patterns altered with subtractive noise. In the next three experiments we reduced the number of associations used to train the MAAM $W_{XY}$ in order to determine how much this factor influences the accuracy of the model. As in previous experiments, we analyzed the behavior of the MAAM $W_{XY}$ under different kind of noises but also using different numbers of associations to train the model.

In Fig. 3 we show the behavior of the MAAM $W_{XY}$ under different type of noises when the model is trained with 20 associations.

For the case of additive, mixed and Gaussian noise (Figs. 3(a, c and d)), we can observe that if only the 2% of the pixels are altered, the MAAM $W_{XY}$ was capable to correctly recall only 17%, 22.2% and 40.1% of the pixels, respectively. For the case of subtractive noise (Fig. 3(b)), we can observe that even when the 90% of the pixels are altered, the MAAM $W_{XY}$ was capable to correctly recall 86.2% of the pixels. In average, for the case of the image patterns altered with additive noise the MAAM $W_{XY}$ 7.13% of the pixels were recalled. For the case of subtractive noise the MAAM $W_{XY}$ in average 92.7% of the pixels were recalled. In short, for the case of mixed and Gaussian noise, the MAAM $W_{XY}$ recalled 7.76% and 11.94%, respectively.

Fig. 4 shows the behavior of the MAAM $W_{XY}$ under different type of noises when the model is trained with 10 associations.

For the case of additive, mixed and Gaussian noise (Figs. 4(a, c and d)), we can observe that if only 2% of the pixels are altered, the MAAM $W_{XY}$ was capable to correctly recalling only 12.9%, 16.4% and 26.5% of the pixels, respectively. For the case of subtractive noise (Fig. 4(b)), we can observe that even when the 90% of the pixels are altered, the MAAM $W_{XY}$ was capable of correctly recalling 94.5% of the pixels. In average, for the case of the image patterns altered with additive noise the MAAM $W_{XY}$ recalled 4.9% of the pixels. For the case of subtractive noise the MAAM $W_{XY}$ in average recalled 97.1% of the pixels. Finally, for the case of mixed and Gaussian noise, the MAAM $W_{XY}$ recalled 5.6% and 10.4%, respectively.

In short, we can say that the accuracy of the MAAM $W_{XY}$ increases as the number of associations decreases. This fact holds only for patterns altered with subtractive noise. For the other kind of noises tested in this set of experiments, the accuracy decreases.

The general behavior of the MAAM $W_{XY}$ is shown in Fig. 6, where clearly we can observe the robustness of this memory with patterns altered with subtractive noise. Although we already know that the MAAM $W_{XY}$ is robust to subtractive noise, nobody had reported results using color images. These results are acceptable and support the applicability of this MAAM $W_{XY}$ to restore true-color images from noisy versions altered with subtractive noise. On the other hand, we expected the worst results for the case of mixed and Gaussian noise, however, the worst result was obtained with the images altered with additive noise. In this experiment we can observe that the MAAM $W_{XY}$ was more robust to Gaussian and mixed noise than additive noise.
Fig. 4. Behavior of the MAAM \( W_{\text{XY}} \) under different type of noises and 10 associations.

Fig. 5. Behavior of the MAAM \( W_{\text{XY}} \) under different type of noises and 5 associations.
This result is interesting because the authors in [15] claim that MAAM are not robust under mixed noise, only for additive and subtractive noise.

### 3.2. Hetero-associative version of $W_{XY}$

The MHAM $W_{XY}$ was not capable to recall the complete set of associations. Even when patterns were not altered with noise, the MHAM $W_{XY}$ correctly recalled only 77.6% of the pixels.

The behavior of the MHAM $W_{XY}$ under different types of noises when the associative model is trained with 20 associations is shown in Fig. 7. For the case of additive noise (Fig. 7(a)), we can observe that if only 2% of the pixels are altered, the MHAM $W_{XY}$ was capable of correctly recalling only 2.2% of the pixels. For the case of mixed and Gaussian noise (Figs. 7(c and d)), we can observe that if only 2% of the pixels are altered, the MHAM $W_{XY}$ was capable of correctly recalling only 3.4% and 17.7% of the pixels, respectively. These percentages decrease when the number of altered pixels increases. For the case of subtractive noise (Fig. 7(b)), we can observe that even when the 90% of the pixels are altered, the MHAM $W_{XY}$ was capable of correctly recalling 73.8% of the pixels. In average, for the case of the image patterns altered with additive noise the MHAM $W_{XY}$ in average recalled 1.4% of the pixels. For the case of subtractive noise the MHAM $W_{XY}$ in average recalled 75.9% of the pixels. Finally, for the case of mixed and Gaussian noise, the MHAM $W_{XY}$ recalled 1.5% and 4.1%, respectively.

**Fig. 6. General behavior of the MAAM $W_{XY}$ under different type of noises.**

![General behavior of MAAM-0 under additive noise](image1)

![General behavior of MAAM-0 under subtractive noise](image2)

![General behavior of MAAM-0 under mixed noise](image3)

![General behavior of MAAM-0 under gaussian noise](image4)

In Fig. 9 we show the behavior of the MHAM $W_{XY}$ under different types of noises when it is trained with 5 associations. When patterns were not altered with noise, the MHAM $W_{XY}$ correctly recalled only 98.8% of the pixels. For the case of additive, mixed and Gaussian noise (Figs. 8(a, c and d)), we can observe that if only the 2% of the pixels are altered with additive noise, the MHAM $W_{XY}$ was capable to correctly recall only 5.4%, 5.5% and 14.1% of the pixels, respectively. For the case of subtractive noise (Fig. 8(b)), we can observe that even when the 90% of the pixels are altered, the MHAM $W_{XY}$ was capable of correctly recalling 87.8% of the pixels. In average, for the case of the image patterns altered with additive noise the MHAM $W_{XY}$ in average recalled 1.4% of the pixels. For the case of subtractive noise the MHAM $W_{XY}$ in average recalled 88.9% of the pixels. Finally, for the case of mixed and Gaussian noise, the MHAM $W_{XY}$ recalled the 1.6% and 4.1%, respectively.

In short, we can say that the accuracy of the MHAM $W_{XY}$ increases as the number of associations is decreased. This fact holds only for patterns altered with subtractive noise. For the other type of noises tested in this set of experiments, the accuracy decreases.
Fig. 7. Behavior of the MHAM \( W_{CV} \) under different type of noises and 20 associations.

Fig. 8. Behavior of the MHAM \( W_{CV} \) under different type of noises and 10 associations.
Fig. 9. Behavior of the MHAM $W_{xy}$ under different type of noises and 5 associations.

Fig. 10. General behavior of the MHAM $W_{xy}$ under different type of noises.
The general behavior of the MHAM \( \text{MHAM}_{\text{WXY}} \) is shown in Fig. 10, where clearly we can observe the robustness of this memory using patterns altered with subtractive noise. However, the results were not as good as in the auto-associative version.

4. Behavior of \( \text{MXY} \) under true-color noisy patterns

In this section, a study of the behavior of \( \text{MXY} \) under true-color noisy patterns is presented. Two type of experiments will be performed: in the first case we study the auto-associative version of \( \text{MXY} \), and then the hetero-associative version.

For the case of the auto-associative version, first to all, we verified if the MAAM \( \text{MAAM}_{\text{MXY}} \) was capable of recalling the complete set of associations. Then we verified the behavior of \( \text{MXY} \) using noisy versions of the images used to train the MAAM. After that, we performed a study of how the number of associations influence the behavior of the MAAM \( \text{MAAM}_{\text{MXY}} \).

For the case of the hetero-associative version, first to all, we verified if the MHAM \( \text{MHAM}_{\text{MXY}} \) was capable to recall the complete set of associations. At last, we verified the behavior of \( \text{MXY} \) using noisy versions of the images used to train the MHAM.

The benchmark used in this set of experiments was the same used in the previous section. Before the MAAM \( \text{MAAM}_{\text{MXY}} \) was trained, each image was transformed into an image pattern. To build an image pattern from the bmp file, the image was read from left-right and up-down; each RGB pixel (hexadecimal value) was transformed into a decimal value and finally, this information was stored into an array.

Once trained the associative memory, we proceed to evaluate the behavior of the two MAAM \( \text{MXY} \) versions.

4.1. Auto-associative version of \( \text{MXY} \)

Even using true-color patterns the MAAM \( \text{MAAM}_{\text{MXY}} \) was capable to recall the complete set of associations.

Fig. 11 shows the behavior of the MAAM \( \text{MAAM}_{\text{MXY}} \) under different types of noises. For the case of subtractive, mixed and Gaussian noise (Figs. 11(b–d)), we can observe that if only the 2% of the pixels are altered, the MAAM \( \text{MAAM}_{\text{MXY}} \) was capable of correctly recalling only 66.9%, 72.4% and 83.8% of the pixels, respectively. These percentages decrease as the number of altered pixels increases. For the case of additive noise (Fig. 11(a)), we can observe that even when the 90% of the pixels are altered, the MAAM \( \text{MAAM}_{\text{MXY}} \) was capable of correctly recalling 59.8% of the pixels. For the case of the image patterns altered with additive noise the MAAM \( \text{MAAM}_{\text{MXY}} \) in average recalled 77.3% of the pixels. For the case of subtractive noise the MAAM \( \text{MAAM}_{\text{MXY}} \) in average recalled 32.8% of the pixels. Finally, for the case of mixed and Gaussian noise, the MAAM \( \text{MAAM}_{\text{MXY}} \) recalled 35.4% and 45.02%, respectively.

In the previous experiment we stored 40 associations and we obtained an accuracy of 77.3% for the case of the MAAM \( \text{MAAM}_{\text{MXY}} \) with patterns altered with subtractive noise. In the next three experiments we reduced the number of associations used to train the MAAM \( \text{MAAM}_{\text{MXY}} \) in order to determine how much this factor influenced in the accuracy of the model.

Fig. 12 shows the behavior of the MAAM \( \text{MAAM}_{\text{MXY}} \) under different types of noises when the model is trained with 20 associations. For the case of subtractive, mixed and Gaussian noise (Figs. 12(b and d)), we can observe that if only 2% of the pixels are altered, the MAAM \( \text{MAAM}_{\text{MXY}} \) was capable of correctly recalling only 51.9%, 59.4% and 75.5% of the pixels, respectively. For the case of additive noise (Fig. 12(a)), we can observe that even when the 90% of the pixels are altered, the MAAM \( \text{MAAM}_{\text{MXY}} \) was capable of correctly recalling 99.5% of the pixels.

Fig. 11. Behavior of the MAAM \( \text{MAAM}_{\text{MXY}} \) under different type of noises and 40 associations.
Fig. 12. Behavior of the MAAM $\text{M}_{\text{MA}}$ under different type of noises and 20 associations.

Fig. 13. Behavior of the MAAM $\text{M}_{\text{MA}}$ under different type of noises and 10 associations.
For the case of the image patterns altered with additive noise the MAAM $\text{M}_{\text{XY}}$ in average recalled 82.9% of the pixels. For the case of subtractive noise the MAAM $\text{M}_{\text{XY}}$ in average recalled 22.1% of the pixels. Finally, for the case of mixed and Gaussian noise, the MAAM $\text{M}_{\text{XY}}$ recalled 24.6% and 35.6%, respectively.

The behavior of the MAAM $\text{M}_{\text{XY}}$ under different types of noises when it is trained with 10 associations is shown in Fig. 10. For the case of subtractive, mixed and Gaussian noise (Figs. 13(b and d)), we can observe that if only 2% of the pixels are altered the MAAM $\text{M}_{\text{XY}}$ was capable of correctly recalling only 40%, 45.6% and 65.5% of the pixels, respectively. For the case of additive noise (Fig. 13(a)), we can observe that even when the 90% of the pixels are altered, the MAAM $\text{M}_{\text{XY}}$ was capable of correctly recalling 83.1% of the pixels.

For the case of the image patterns altered with additive noise, the MAAM $\text{M}_{\text{XY}}$ in average recalled 91.1% of the pixels. For the case of subtractive noise the MAAM $\text{M}_{\text{XY}}$ in average recalled 14.1% of the pixels. Finally, for the case of mixed and Gaussian noise, the MAAM $\text{M}_{\text{XY}}$ recalled 16.9% and 31.7%, respectively.

Fig. 14 shows the behavior of the MAAM $\text{M}_{\text{XY}}$ under different types of noises when the model is trained with 5 associations. For the case of subtractive, mixed and Gaussian noise (Figs. 14(b and d)), we can observe that if only 2% of the pixels are altered, the MAAM $\text{M}_{\text{XY}}$ was capable of correctly recalling only 6.7%, 14.5% and 37.6% of the pixels, respectively. For the case of additive noise (Fig. 14(a)), we can observe that even when the 90% of the pixels are altered, the MAAM $\text{M}_{\text{XY}}$ was capable of correctly recalling 99.9% of the pixels.

In average for the case of the image patterns altered with additive noise the MAAM $\text{M}_{\text{XY}}$ recalled 98.9% of the pixels. For the case of subtractive noise the MAAM $\text{M}_{\text{XY}}$ in average recalled 1.7% of the pixels. Finally, for the case of mixed and Gaussian noise, the MAAM $\text{M}_{\text{XY}}$ recalled 2.1% and 7.1%, respectively.

In short, we can say that the accuracy of the MAAM $\text{M}_{\text{XY}}$ increases as the number of associations is decreased. This fact holds only for patterns altered with additive noise. For the other type of noises tested in this set of experiments, the accuracy decreases.

The general behavior of the MAAM $\text{M}_{\text{XY}}$ is shown in Fig. 15, where clearly we can observe the robustness of this memory with patterns altered with additive noise. Although we already know that the MAAM $\text{M}_{\text{XY}}$ is robust to additive noise, nobody had reported results using color images. These results are acceptable and support the applicability of this MAAM $\text{M}_{\text{XY}}$ to reconstruct true-color images from noisy versions altered with additive noise. On the other hand, we expected the worst results for the case of mixed and Gaussian noise, however, the worst result was obtained with the images altered with subtractive noise. In this experiment we could observe that the MAAM $\text{M}_{\text{XY}}$ was more robust to Gaussian and mixed noise than subtractive noise. This result is interesting because the authors in [15] claim that MAAM are not robust under mixed noise, only for additive and subtractive noise.

### 4.2. Hetero-associative version of $\text{M}_{\text{XY}}$

The MHAM $\text{M}_{\text{XY}}$ was not capable to recall the complete set of associations. Even when patterns were not altered with noise, the MHAM $\text{M}_{\text{XY}}$ correctly recalled only 43.1% of the pixels.

Fig. 16 shows the behavior of the MHAM $\text{M}_{\text{XY}}$ under different types of noises when it is trained with 20 associations. For the case of subtractive, mixed and Gaussian noise (Figs. 16(b–d)), we can observe that if only the 2% of the pixels are altered, the MHAM $\text{M}_{\text{XY}}$ was capable to correctly recall only the 7.2%, 10% and 22.6% of the pixels, respectively. These percentages decrease when the number of altered pixels increases. For the case of additive noise (Fig. 16(a)), we can observe that even when the 90% of the pixels are altered, the
MHAM $M_{XY}$ was capable of correctly recalling 39.5% of the pixels. In average, for the case of the image patterns altered with additive noise the MHAM $M_{XY}$, in average recalled 41.3% of the pixels. For the case of subtractive noise the MHAM $M_{XY}$ in average recalled 1.2% of the pixels. Finally, for the case of mixed and Gaussian noise, the MHAM $M_{XY}$ recalled 1.8% and 6.7%, respectively.

Fig. 15. General behavior of the MAAM $M_{XY}$ under different type of noises.

Fig. 16. Behavior of the MHAM $M_{XY}$ under different type of noises and 20 associations.
Fig. 17 shows the behavior of the MHAM $M_{XY}$ under different type of noises when trained with 10 associations. When patterns were not altered with noise, the MHAM $M_{XY}$ correctly recalled only 67.3% of the pixels. For the case of subtractive, mixed and Gaussian noise (Figs. 17(b–d)), we can observe that if only 2% of the pixels are altered, the MHAM $M_{XY}$ was capable of correctly...
recalling only 11.5%, 17.3% and 37% of the pixels, respectively. For the case of additive noise (Fig. 17(a)), we can observe that even when the 90% of the pixels are altered, the MHAM \( \text{MHAM}_{XY} \) was capable of correctly recalling 61.3% of the pixels. In average, for the case of the image patterns altered with additive noise the MHAM \( \text{MHAM}_{XY} \) recalled 65% of the pixels. For the case of subtractive noise...

Fig. 19. General behavior of the MHAM under different type of noises.

Fig. 20. General behavior of the MAAM altering by adding and subtracting units to the value of all pixels.
the MHAM $\text{MHAM}_{\text{XY}}$ in average recalled 1.7% of the pixels. Finally, for the case of mixed and Gaussian noise, the MHAM $\text{MHAM}_{\text{XY}}$ recalled 2.6% and 9.6%, respectively.

Fig. 18 shows the behavior of the MHAM $\text{MHAM}_{\text{XY}}$ under different type of noises when the model is trained with 5 associations. When patterns were not altered with noise, the MHAM $\text{MHAM}_{\text{XY}}$ correctly recalled only 96.1% of the pixels. For the case of subtractive, mixed and Gaussian noise (Figs. 18(b–d)), we can observe that if only the 2% of the pixels are altered, the MHAM $\text{MHAM}_{\text{XY}}$ was capable of correctly recalling only 5.3%, 10.7% and 38.1% of the pixels, respectively. For the case of additive noise (Fig. 18(a)), we can observe that even when the 90% of the pixels are altered, the MHAM $\text{MHAM}_{\text{XY}}$ was capable of correctly recalling 93.5% of the pixels. In average, for the case of the image patterns altered with additive noise the MHAM $\text{MHAM}_{\text{XY}}$ recalled 95% of the pixels. For the case of subtractive noise the MHAM $\text{MHAM}_{\text{XY}}$ in average recalled 1.2% of the pixels. Finally, for the case of mixed and Gaussian noise, the MHAM $\text{MHAM}_{\text{XY}}$ recalled 1.4% and 4.8%, respectively.

In short, we can mention that the accuracy of the MHAM $\text{MHAM}_{\text{XY}}$ increases as the number of associations decreases. This fact holds only for patterns altered with additive noise. For the other type of noise tested in this set of experiments, the accuracy decreases.

The general behavior of the MHAM $\text{MHAM}_{\text{XY}}$ is shown in Fig. 19, where clearly we can observe the robustness of this memory with patterns altered with additive noise. However the results were not as good as in the auto-associative version.

5. Robustness of MAMs under other image transformations

Until now, we have analyzed the behavior of the model when images are altered with noise. However, it is also important to
analyze the model with other kind of noises. In this section we analyze the accuracy of the MAAM \( W_{XY} \) with some simple images transformations: increasing and/or decreasing the value of all pixels from the image.

As in previous experiments, before we train the MAAM \( W_{XY} \) with 40 associations, we transform each image into an image pattern, and then we proceed to evaluate the accuracy of the model using the same benchmark but adding one unit to the value of all pixels of the image, then two units and so on until 90 units.

The accuracy of the model is shown in Fig. 20. As can be appreciated, while the results in terms of the NMSE indicate that the model is robust to this kind of transformation, the results in terms of the percentage of recalled pixels indicated that the memory recalled 0% of the original pixels. From a strict point of view, the model is not robust to this kind of transformation, but from an image processing point of view the model is robust with an average error of less than 0.5. It is also important to remark that the error increases as the amount of noise added to all pixel is increased.

6. Image categorization using MAMs

Image categorization is not trivial when pictures are taken from real life situations. This implies that categorization must be invariant to several image transformations such as translations, rotations, scale changes, illumination changes, orientation changes, noise, and so on [21].

In this section we describe how images can be categorized using the MAM already described and the methodology introduced in [21]. Suppose that we feed a MAM with a picture and we expect that it responds with something indicating the content of the picture. If, for example the picture contains a lion, we would expect that the MAM should respond with the word “lion”.

A first step to solve this problem was reported in [21]; now we will use it to show the applicability of the MAM providing a solution to this problem when the concerned images are distorted only by additive noise.

Following the procedure described in [21] we firstly selected a set of images, in this case the benchmark used in previous experiment. Then, we associated these images with describing words. The images and the describing words are our fundamental set of associations, with \( x^k \) is the \( k \)-image and \( y^k \) the \( k \)-describing word. With this set of associations we proceed to train the MAM.

Either for learning or recall, each color image \( f_k(i,j) \) is transformed into an image pattern. If image \( f_k(i,j) \) is of size \( M \times N \) pixels, then its corresponding vector is \( x^k = [x^k_1 \ x^k_2 \ ... \ x^k_{MN}]^T \). The elements \( y^r, r = 1, \ldots, R \) of vector \( y^k \) correspond to the ASCII codes of the letters of each describing word, where \( R \) is the number of letters of a given word.

In Fig. 21, we show the information used to train the associative memory. By using this set of association we expect that when feeding the MAM with the image which contains an agapanthus, we will recall the word “agapanthus”, even if the image is altered with additive noise. As the reader can guess, we will train a MHAM \( M_{XY} \). Once trained the associative model, we proceed to test the accuracy of the proposal altering the images with additive noise.
As you can appreciate in Fig. 22, not all the 14,400 images were correctly categorized, in other words, not all the words associated with the images were correctly recalled. In average, the accuracy of the proposal in this image categorization task was of 82.6%. Something important to notice is that even if the noise added to the images increases, the accuracy of the model remains almost equal.

7. Image restoration using MAMs

In this section we describe how the associative model already exposed can be applied to the problem of image restoration. Particularly, we will show the applicability of the model in the restoration of several \((k)\) famous paintings.

Each painting was first digitalized to a RGB format of size \(250 \times 250\) pixels. Something important to remember is that for memory reasons so larger images cannot be stored in the described model. For this reason we propose to split each painting into \(c\) sub-images (refer to Fig. 23). We then use these sub-images to train \(k\) associative memories (one per each painting). Once trained the \(k\) associative memories, in order to restore a distorted version of a painting, we split the incomplete painting into \(c\) sub-images and then, we operate the corresponding associative memory using these sub-images. Once recalled the \(c\) sub-images, we merged the information to generate the complete restored painting.

In order to test the accuracy of this methodology, we have used a benchmark of images of seven famous paintings. Fig. 24 (second column) shows the images of the seven chosen paintings used to train the described model. Once trained the associative memories by means of the sub-images obtained from each painting, we proceeded to test the accuracy of the model by

<table>
<thead>
<tr>
<th>Original painting</th>
<th>Incomplete painting</th>
<th>% of information removed</th>
<th>Restored painting</th>
<th>NMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Barque of Dante (Eugène Delacroix)</td>
<td><img src="image1" alt="Image" /></td>
<td>8.9%</td>
<td><img src="image2" alt="Image" /></td>
<td>0.0053</td>
</tr>
<tr>
<td>Wanderer above the Sea of Fog (Caspar David Friedrich)</td>
<td><img src="image3" alt="Image" /></td>
<td>13.44%</td>
<td><img src="image4" alt="Image" /></td>
<td>0.0041</td>
</tr>
<tr>
<td>Arab Horses Fighting in a Stable (Eugène Delacroix)</td>
<td><img src="image5" alt="Image" /></td>
<td>12.92%</td>
<td><img src="image6" alt="Image" /></td>
<td>0.0026</td>
</tr>
<tr>
<td>Last Judgment (Michelangelo)</td>
<td><img src="image7" alt="Image" /></td>
<td>18.9%</td>
<td><img src="image8" alt="Image" /></td>
<td>0.0113</td>
</tr>
<tr>
<td>The Raft of the Medusa (Théodore Géricault)</td>
<td><img src="image9" alt="Image" /></td>
<td>25.62%</td>
<td><img src="image10" alt="Image" /></td>
<td>0.0025</td>
</tr>
<tr>
<td>La Gioconda (Leonardo da Vinci)</td>
<td><img src="image11" alt="Image" /></td>
<td>28.75%</td>
<td><img src="image12" alt="Image" /></td>
<td>0.0137</td>
</tr>
<tr>
<td>The Massacre at Chios (Eugène Delacroix)</td>
<td><img src="image13" alt="Image" /></td>
<td>23.59%</td>
<td><img src="image14" alt="Image" /></td>
<td>0.0045</td>
</tr>
</tbody>
</table>

Fig. 24. Original images (2nd column). Distorted versions (3rd column). % of information removed (4th column). Restored images (5th column). NMSE by a MAM (6th column).
using a distorted version of the original paintings. Incomplete versions with the corresponding percentage of information removed from the original images are shown in columns three and four of Fig. 24.

Finally, the corresponding restored paintings with their corresponding NMSE are shown in column five and six of Fig. 24. The NMSE gives the normalized mean square error between to images, expressed as:

$$NMSE(x,y) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij} - y_{ij})^2}{\sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij})^2}$$

where $x$ is the original image and $y$ is the restored image.

From the above experiments we can observe that the results provided by this kind of associative model are highly encouraging and suggest that this kind of models can be applied to provide solutions to other practical problems such as image restoration. As you can appreciate from Fig. 25, the NMSE is too small which indicates how the associative memories succeed in restoring images.

One question that remains in the solution of this problem is the post-processing of the restored images trying to partially or totally eliminate the residual visible distortions (fifth column of Fig. 24). It is well-known that this is not possible with MAMs, due to the operators involved are idempotent. i.e. if we present the restored images (fifth column of Fig. 24) to the corresponding MAMs, the same results will be obtained. Nowadays, we are looking for new associative models that allow to iteratively restoring images until getting desired results.

8. Conclusions

In this paper, a complete study of the behavior of the morphological associative memory in the reconstruction of true-color images using a benchmark of 14400 images altered by different type of noises was presented.

Due to this associative model had been only applied to binary and gray level patterns; this paper is useful to really understand the power and limitations of this model. Two types of experiments were performed. In the first case we studied the auto-associative version of MAM, and then the hetero-associative version.

In both cases we verified if the MAM was capable to recall the complete set of associations. Then we verified the behavior of MAM using noisy versions of the images used to train it. After that, we performed a study of how the number of associations influences the behavior of the MAM.

Through several experiments, we found some interesting properties of this associative model. While MAAMs present perfect recall, MHAMs do not present perfect recall. Furthermore, MHAMs are not sensitive to the amount of noises; while MAAMs decreases its accuracy when the level of noise is increased, MHAMs holds the accuracy even when the noise is increased.

As we already knew, MAAM $M_{x,y}$ is more robust to additive noises than the other type of noises. However, MAAM $M_{x,y}$ is more robust to Gaussian and mixed noise than subtractive noise. For the case of MAAM $W_{x,y}$, this memory is more robust to subtractive noises than the other type of noises. However, MAAM $W_{x,y}$ is more robust to Gaussian and mixed noise than subtractive noise.

Regarding to the storage capacity, we have found that the accuracy of the model is too sensitive to the number of associations stored in the MAM. In general we can say that as the number of association is increased, the accuracy of the memory decreases when patterns are altered with noise to which they are more robust. If patterns are altered with noise to which they are not robust, the accuracy of the memory tends to increase as the number of associations is increased.

On the other hand, while the hetero-associative version holds almost the same accuracy when the amount of additive noise is increased, the accuracy of the auto-associative memory decreases. In Figs. 25 and 26, the general behavior of the four associative
memory versions is presented. As can be observed from Fig. 25, the best accuracy is provided by \( W_{XY} \) in both cases (auto- and hetero-associative version) when patterns are altered by subtractive noise. In average, MAAM \( M_{XY} \) correctly recalled 87.5% of the pixels when patterns were altered by additive noise. MHAM \( M_{XY} \) correctly recalled 67.1% of the pixels when patterns were altered by additive noise. MAAM \( W_{XY} \) correctly recalled 94.2% of the pixels when patterns were altered by additive noise. MHAM \( W_{XY} \) correctly recalled 87.7% of the pixels when patterns were altered by additive noise.

In Fig. 26 it is shown the behavior the morphological associative model in terms of the normalized mean square error. In addition we have tested the accuracy of the MAM in the content of image categorization and image restoration problems. The results provided by the model were highly acceptable. This set of experiments presented supports the robustness and usefulness of the morphological associative model in different kind of situations.

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